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Comparative Analysis of Buckling Behaviour of Cylindrical Shells Reinforced with Inclined Stiffeners and that Reinforced with Rings-and-Stringers under Uniform Bending

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Abstract—This research presents comparative analysis of buckling behaviour of cylindrical shells reinforced with inclined stiffeners and that reinforced with rings-and-stringers under Uniform Bending. The method of solution was carried out by the use of nonlinear large deflection theory and the effect of initial imperfections in the strain-displacement equations was considered. The Ritz method was used to determine the buckling stress parameter of the shell. Numerical examples were carried by varying the angle of inclination of the stiffeners at different imperfect ratios with other properties like: flexural rigidity and torsional rigidity of the stiffeners, deflection parameters, internal pressure and radius of curvature of the shell being kept constant. The results shows that the worst critical buckling stress of internally pressurized thin cylindrical shell considered in this research occurs when the stiffeners are inclined at 450 at imperfect ratio of 0.1 While, the maximum buckling stress occurs when the stiffeners are inclined at 100 at imperfect ratio of 0.5 .Also, the buckling stress for the cylinders reinforced with rings-and-stringers considered in this work are averagely greater than those reinforced with stiffeners inclined at 450, but less than those reinforced with stiffeners inclined at 100, 200, 300, 500 and 600 respectively. Thus, rings-and-stringers are more effective than inclined stiffeners at 450, but less effective than stiffeners inclined at 100, 200, 300, 500 and 600 respectively.

Index Terms—Thin, cylindrical shell, buckling, stress, uniform bending, the Ritz, imperfect ratio, nonlinear, deflection theory, inclined stiffeners, rings-and-stringers.

1 Introduction

Buckling is often critical in thin-walled or light weight members such as slender columns, plates and cylindrical shells which are subjected to predominantly compressive action (Iyengar, 1988).

Buckling of cylindrical shells can occur when the structure is subjected to the individual or combined action of axial compression, external pressure and torsion (Houliara, 2008; Catellani et al, 2004). Its buckling behaviour is not accurately predicted by linear elastic equations due to initial imperfections of the shell structure under the action of compressive loads. Such imperfections may be either geometrical imperfections (for example: out of straightness, initial ovality, dents, swells, circularity, cylindricity and geometrical eccentricities); structural imperfections (i.e. residual stresses and material inhomogenities) or loading imperfections (i.e. non uniform edge load distribution, unintended edge moments, load eccentricities and load alignments as well as imperfect boundary conditions) (George, 1996). Also the constructional defects, such as small holes, cut-outs, rigid inclusions and delamination could be regarded as structural imperfections. Out of all these imperfections the geometrical imperfections are more dominant in determining the load carrying capacity of thin cylindrical shells (Zhang and Han, 2007).

Cylindrical shells in engineering structures with large aspect ratios are typically stiffened against buckling by circumferential and longitudinal members known as ring and stringers stiffeners respectively. The use of the stiffeners improves the resistance of cylindrical shells to buckling (George, 1996; Arani et al, 2007).

In this work, the buckling behaviour of thin cylindrical shells reinforced with inclined stiffeners and that reinforced with rings-and-stringers were compared. The analysis was done by using Ritz method. The results obtained from the analysis will enable designers of thin cylindrical shell structures to select suitable reinforcement for the structure.

2 STRAIN ENERGY EXPRESSION FOR THE CYLINDRICAL SHELL

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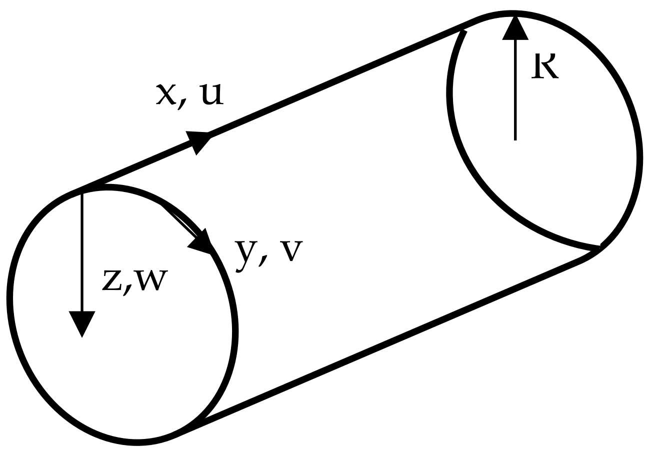

Fig. 1: Coordinates and Displacement Components of a point on the Middle- surface of the shell

let x and y be the axial and circumferential axis in the median surface of the undeformed cylindrical shell as shown in Fig. 1, w is the total radial deflection and w_0 represents the initial radial deflection.

From the theory of elasticity, the strain – displacement relations of the cylindrical shell is as expressed in Eqns. (1a), (1b) and (1c) respectively

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{dw_0}{\partial x}\right)^2 \quad (1a)$$

$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \frac{w - w_0}{R} \quad (1b)$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\cdot\frac{\partial w_0}{\partial y} \quad (1c)$$

The stresses and strains in the middle surface of the shell in the case of plane stress are related to each other by the following equations.

$$\sigma_x = \frac{E}{1-\mu^2}(\epsilon_x + \mu\,\epsilon_y) \quad (2a)$$

$$\sigma_y = \frac{E}{1-\mu^2}(\epsilon_y + \mu\,\epsilon_x) \quad (2b)$$

$$\sigma_{xy} = \frac{E}{2(1+\mu)}\epsilon_{xy} \quad (2c)$$

Substituting Eqns. (1a), (1b) and (1c) into their related equations in Eqns. (2a), (2b) and (2c), the followings were obtained;

$$\sigma_x = \frac{E}{1-\mu^2}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \mu\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right)\right]\right\} \quad (3a)$$

$$\sigma_y = \frac{E}{1-\mu^2}\left\{\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right) + \mu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]\right\} \quad (3b)$$

$$\sigma_{xy} = \frac{E}{2(1-\mu^2)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}\cdot\frac{\partial w_0}{\partial y}\right] \quad (3c)$$

For plane stress state, the non-zero components of stress tensor, $\sigma_x, \sigma_y, \sigma_{xy}$ satisfied the following equilibrium using Airy stress function F.

$$\sigma_x = \frac{\partial^2 F}{\partial y^2};\ \sigma_y = \frac{\partial^2 F}{\partial x^2};\ \sigma_{xy} = \frac{-\partial^2 F}{\partial x \partial y} \quad (4)$$

Eliminating variables $\boldsymbol{u}$ and $\boldsymbol{v}$ in Eqns. (3) and (4), the relation between stress function F and radial component displacement, $\boldsymbol{w}$ was expressed as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial x^2}\cdot\frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R}\frac{\partial^2 w_0}{\partial x^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right] \quad (5a)$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is called Laplace operator.

$$(\nabla^2)^2 F = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial x^2}\cdot\frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R}\frac{\partial^2 w_0}{\partial x^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right] \quad (5b)$$

5 For simplicity, w was assumed to be proportional to w_0. Thus,

$$Л = \frac{w_0}{w} \quad (6)$$

Where $Л$ is called imperfection ratio and it is independent of x and y.

With the expression from Eqns (5b) and (6), the compatibility equation was expressed as;

$$\left(\frac{1}{1-Л}\right)\nabla^4 F = E(1+Л)\left[\left(\frac{\partial^2 w}{\partial x \partial y^2}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}\right] - \frac{E}{R}\frac{\partial^2 w}{\partial x^2} \quad (7)$$

Where ∇^4 is called Bilharmonic operator.

Equation (7) is the compatibility equation of perfect thin cylindrical shell.

The strain energy of isotropic medium referred to arbitrary orthogonal coordinates was expressed as:

$U = \frac{1}{2}\iiint_{vol} \sigma_{ij}\,\epsilon_{ij}\,dvol = \frac{1}{2}\iiint_{vol}\left[\sigma_x\epsilon_x + \sigma_y\epsilon_y + \sigma_{xy}2\epsilon_{xy} + \sigma_{xz}2\epsilon_{xz} + \sigma_{yz}2\epsilon_{yz}\right]dxdydz$ (8a)

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Substituting Eqns. 1(a-c), 2(a-c), 3(a-c) and 4 into Eqn. (8a), we have expressions stated in Eqns. (8) and (9) respectively:

i. *The extensional strain energy in the shell was expressed as;*

$$U_e = \frac{h}{2E}\int_0^L \int_0^{2\pi R} \left\{ \left(\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} \right)^2 + 2(1+\mu)\left[\left(\frac{\partial^2 F}{\partial x \partial y} \right)^2 - \frac{\partial^2 F}{\partial x^2} \cdot \frac{\partial^2 F}{\partial y^2} \right] \right\} dxdy \quad (8)$$

ii. *The potential due to the internal pressure, p*

$$U_p = \int_0^L \int_0^{2\pi R} p(w - w_0)\, dxdy \qquad (9)$$

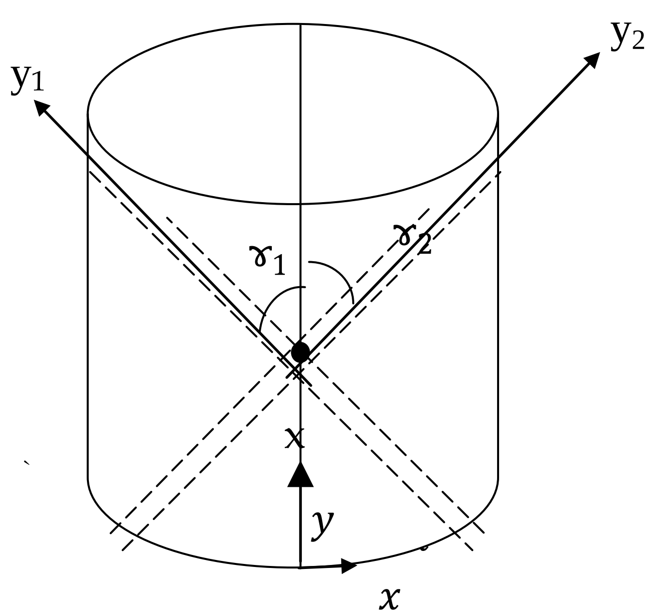
iii. *The potential due to edge bending of the shell*

As a result of the eccentric loading of the shell, the potential due to the edge bending of the shell is the product of applied bending force and the length in the direction of bending. This expressed as:

$$U_m = \frac{-\sigma_b h}{E}\int_0^L \int_0^{2\pi R} \left[cos\frac{y}{R} \left\{ \left(\frac{\partial^2 F}{\partial y^2} - \mu \frac{\partial^2 F}{\partial x^2} \right) - \frac{1}{2}\left(\frac{\partial w}{\partial x} \right)^2 + \frac{1}{2}\left(\frac{\partial w_0}{\partial x} \right)^2 \right\} \right] dxdy \quad (10)$$

Where σ_b = applied peak bending stress

iv. *The bending strain energy of stiffeners.* Considering Fig. 2, the stiffeners were assumed parallel with they_1, y_2 coordinates lines and the principal direction of the cylindrical shell coincide with x, y, lines**.**



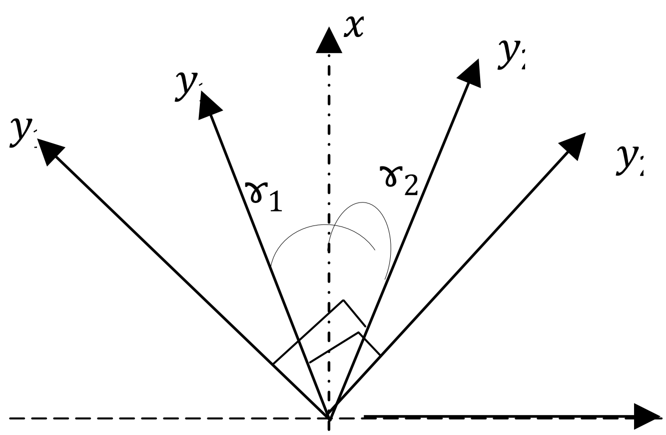


Fig. 2: The coordinate system of the stiffeners of the cylindrical shells and stiffeners

The subscript k was used for k^{th}stiffnener, which is inclined at an angle, γ_1 with generator of the cylinders and is parallel with y_1-line and normal to y_2'– line. Hence, the bending strain energy in the k^{th}stiffener is

$$U_{b,k} = \sum_{k=1}^{N_k} \frac{E_k I_k}{2} \int_0^{L_k} \left(\frac{\partial^2 w}{\partial y_1^2} - \frac{\partial^2 w_0}{\partial y_1^2} \right)^2_{y_2'=0} dy_1 \quad (11)$$

Where $N_k$denotes the number of the stiffeners in γ_1 – direction.

$E_k I_k$represents the flexural rigidity of the k^{th}stiffener. The limit L_k is the length of the stiffener in γ_1 – direction.

Similarly, the bending strain energy in the j^{th}stiffener which is parallel with y_2 – line and nomal to y_1' - line as shown in Fig. 2.

$$U_{b,j} = \sum_{j=1}^{N_j} \frac{E_j I_j}{2} \int_0^{L_j} \left(\frac{\partial^2 w}{\partial y_2^2} - \frac{\partial^2 w_0}{\partial y_2^2} \right)^2_{y_1'=0} dy_2 \quad (12)$$

The subscript j was used for j^{th} stiffener which is inclined at angle of γ_2 with the generator of the cylinder. Where N_j is the number of the stiffeners in γ_2 – direction.

$E_j I_j$ represents the flexural rigidity of the j^{th} stiffeners. The limit L_j is length of the stiffener in γ_2 – direction.

i. *The torsion strain energy of the $k^{th}$and j^{th} stiffeners were*

$$U_{T,k} = \sum_{k=1}^{N_k} \frac{G_k J_k}{2} \int_0^{L_k} \left[\frac{\partial^2 (w - w_0)}{\partial y_1 \partial y_2'} \right]^2_{y_2'=0} dy_1 \qquad (13)$$

$$U_{T,j} = \sum_{j=1}^{N_j} \frac{G_j J_j}{2} \int_0^{L_j} \left[\frac{\partial^2 (w - w_0)}{\partial y_1 \partial y_2'} \right]^2_{y_1'=0} dy_2 \qquad (14)$$

Where G J represents the torsional rigidity of stiffeners, with subscript j representing stiffeners in ɤ_2 – direction and subscript k is for stiffeners in ɤ_1- direction. In this analysis, the inclined angles, ɤ_1 and ɤ_2 are considered in axial symmetry for inclined stiffeners.

The deflection shape of the cylindrical shell under uniform bending was assumed as:

$$w = f_1 + cos^2({}^{y}/_{2R})\left[f_2 \cos\frac{mx}{R} cos\frac{ny}{R} + f_3 cos\frac{2mx}{R} + f_4 \cos\frac{2ny}{R}\right] \quad (15)$$

Where m and n are the numbers of waves in axial and circumferential directions respectively. Using compatibility equation in Eqn (7), the corresponding stress function for cylindrical shell under uniform bending:

$$F = -\frac{\sigma}{2}y^2 + \sigma_b R^2 \cos\frac{y}{R} + \frac{1}{2}\frac{PR}{h}x^2 + a_{11}\cos\frac{mx}{R}\cos\frac{ny}{R} + a_{22}\cos\frac{2mx}{R}\cos\frac{2ny}{R} + a_{20}\cos\frac{2mx}{R} + a_{02}\cos\frac{2ny}{R} + a_{31}\cos\frac{3mx}{R}\cos\frac{ny}{R} + a_{13}\cos\frac{mx}{R}\cos\frac{3ny}{R} \quad (16)$$

Where σ and σ_b are the average axial and peak bending stresses, respectively and are positive for compression.

Substituting Eqn(16) into Eqn(7) and minimizing the resulting equation, the coefficients $a_{11}, a_{22}, a_{02}, a_{20}, a_{31}, a_{13}$ in Eqn. (16) were determined in terms of f_2, f_3, and f_4 as shown in Eqns.17(a-f)

$$\forall^*_{20} = \frac{a_{20}}{Eh^2} = \frac{1}{256}\left[32\text{ᶀ}^*_4\beta(1-\text{Л}) - \left(3\bar{\mu}^2 + \frac{1}{m^2}\right)\text{ᶀ}^{*2}_2(1-\text{Л}^2)\right. \quad (17a)$$

$$\forall^*_{02} = \frac{a_{02}}{Eh^2} = -(1-\text{Л}^2)\frac{3\text{ᶀ}^{*2}_2}{256\bar{\mu}^2} \quad (17b)$$

$$\forall^*_{11} = \frac{a_{11}}{Eh^2} = -\frac{(1-\text{Л}^2)\left[\left(12\bar{\mu}^2 + \frac{1}{m^2}\right)(\text{ᶀ}^*_3 + \text{ᶀ}^*_4)\text{ᶀ}^*_2\right] - 8(1-\text{Л})\beta\text{ᶀ}^*_2}{16(1+\bar{\mu}^2)^2} \quad (17c)$$

$$\bar{\mu} = \frac{n}{m}, \beta = \frac{R}{m^2h}, \quad \text{ᶀ}^*_1 = \frac{f_i}{h}, i = 2,3,4 \quad \bar{\sigma} = \frac{\sigma R}{Eh}$$

$$\forall^*_{22} = \frac{a_{22}}{Eh^2} = -(1-\text{Л}^2)\left[\frac{\frac{\text{ᶀ}^{*2}}{m^2} + \text{ᶀ}^*_3\text{ᶀ}^*_4\left(96\bar{\mu}^2 + {}^{8}/_{m^2}\right)}{256(1+\bar{\mu}^2)^2}\right] \quad (17d)$$

$$\forall^*_{31} = \frac{a_{31}}{Eh^2} = -\frac{\left(12\bar{\mu}^2 + \frac{9}{m^2}\right)}{16(9+\bar{\mu}^2)^2}\text{ᶀ}^*_2\text{ᶀ}^*_4(1-\text{Л}^2) \quad (17e)$$

$$\forall^*_{13} = \frac{\left(12\bar{\mu}^2 + \frac{1}{m^2}\right)}{16(9\bar{\mu}^2+1)^2}\text{ᶀ}^*_2\text{ᶀ}^*_4(1-\text{Л}^2) \quad (17f)$$

Where
$\bar{\mu}$ *is called wavelength ratio in axial and circumferential direction.*

3.0 EXPRESSION OF TOTAL POTENTIAL FOR CYLINDRICAL SHELL WITH INCLINED STIFFENERS SUBJECTED TO INTERNAL PRESSURE AND UNIFORM BENDING

The total potential of the system, $\prod$ is the sum of the strain energy and the potential of the applied loads. Thus,

$$\prod = U_e + U_b + U_m + U_P + U_{b,k} + U_{b,j} + U_{T,k} + U_{T,j} \quad (18)$$

3.1 Minimization of Total Potential Energy, $\bar{\prod}$, of Internally Pressurized of the Thin Cylindrical Shell Reinforced with Inclined Stiffeners

The non-dimensional form of the total potential of the system is express as shown in Eqn (18b)

$$\bar{\prod} = \bar{U}_e + \bar{U}_b + \bar{U}_m + \bar{U}_P + \bar{U}_{b,k} + \bar{U}_{b,j} + \bar{U}_{T,k} + \bar{U}_{T,j} \quad (18b)$$

The total potential energy of the internally pressurized cylindrical shell subjected bending must be a minimum when the structure is in equilibrium. The minimization of the non-dimensional form of the total energy, $\bar{\prod}$ is as expressed in Eqn (19)

$$\frac{\partial\bar{\prod}}{\partial\text{ᶀ}^*_2} = 0, \frac{\partial\bar{\prod}}{\partial\text{ᶀ}^*_3} = 0, \frac{\partial\bar{\prod}}{\partial\text{ᶀ}^*_4} = 0 \quad (19)$$

Evaluation of $\frac{\partial\bar{U}_2}{\partial\text{ᶀ}^*_2} = 0$, $\frac{\partial\bar{U}_2}{\partial\text{ᶀ}^*_3} = 0$, $\frac{\partial\bar{U}_2}{\partial\text{ᶀ}^*_4} = 0$ yielded Eqns (20) - (22):

$$\frac{\bar{\vartheta}_1\beta}{1-\text{Л}} = \bar{A}_1 + \beta^2(\bar{A}_2 + \bar{A}_3\lambda + \bar{A}_4\lambda^2) + \text{ᶀ}^{*2}_2\bar{A}_5 \quad (20)$$

$$\frac{\bar{\vartheta}_2\beta}{1-\text{Л}} = \text{฿}_1 + \beta^2\text{฿}_2\lambda^2 + \text{ᶀ}^{*2}_2\left(\text{฿}_3 + \frac{\text{฿}_4}{\lambda}\right) \quad (21)$$

$$\bar{\vartheta}_3\frac{\beta}{1-\text{Л}} = \bar{d}_1 + (\bar{d}_2 + \bar{d}_3\lambda^2)\beta^2 + \text{ᶀ}^{*2}_2\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right) \quad (22)$$

The notations used in Eqns.(20), (21) *and* (22) were defined as follows;

$$\bar{\vartheta}_1 = \frac{3}{2}\bar{\sigma} + \bar{\sigma}_b - \frac{3\bar{\mu}^2\bar{P}}{2} = \frac{3}{2}\bar{\sigma} + \bar{\sigma}_b + \bar{\vartheta}_2 \quad (23)$$

$$\bar{\vartheta}_2 = -\frac{3\bar{\mu}^2\bar{P}}{2} \quad (24)$$

$$\bar{\vartheta}_3 = \sigma_b + \frac{3}{2}\bar{\sigma} \quad (25)$$

$$\bar{A}_1 = \frac{1}{8\bar{A}_2(1-\mu^2)} + \overleftrightarrow{\psi}_1 \quad (26)$$

$$\bar{A}_2 = \frac{1}{(1+\bar{\mu}^2)^2} \quad (27)$$

$$\bar{A}_3 - \frac{3}{2}\left[(2+\text{Л})(1+\lambda_1)\bar{\mu}^2\bar{A}_2 + \frac{1}{4}\bar{\mu}^2\lambda_1\right] \quad (28)$$

$$\bar{A}_4 = (1+\text{Л})\left\{\frac{9(1+\lambda_1)^2\bar{\mu}^4\bar{A}_2}{4} + \frac{9\bar{\mu}^4}{4}\left[\frac{\lambda_1^2}{(9+\bar{\mu}^2)^2} + \frac{1}{(1+9\bar{\mu}^2)^2}\right]\right\} \quad (29)$$

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$$\bar{A}_5 = (1+Л).\frac{9(1+\bar{\mu}^4)}{256} \quad (30)$$

$$\overleftrightarrow{\psi}_1 = \sum_{k=1}^{Nk} \frac{\bar{E}_k \bar{I}_k \bar{L}_k}{2}\left[\frac{3}{2}(C_1^4 + 6\bar{\mu}^2 C_1^2 S_1^2 + \bar{\mu}^4 S_1^4)\right] + \sum_{j=1}^{N_j} \frac{\bar{E}_j \bar{I}_j \bar{L}_j}{2}\left[\frac{3}{2}(C_2^4 + 6\bar{\mu}^2 C_2^2 S_2^2 + \bar{\mu}^4 S_2^4)\right] + \sum_{k=1}^{Nk} \frac{3\bar{G}_k \bar{J}_k \bar{L}_k}{8}[(C_1 + \bar{\mu}S_1)^2 (S_1 - \bar{\mu}C_1)^2 + (C_1 - \bar{\mu}S_1)^2 (S_1 + \bar{\mu}C_1)^2] + \sum_{j=1}^{N_j} \frac{3\bar{G}_j J_j \bar{L}_j}{8}[(C_2 + \bar{\mu}S_2)^2 (S_2 - \bar{\mu}C_2)^2 + (C_2 - \bar{\mu}S_2)^2 (S_2 + \bar{\mu}C_2)^2] \quad (31)$$

$$\mathbb{B}_1 = \frac{\bar{\mu}^4}{2(1-\mu^2)} + \overleftrightarrow{\psi}_2 \quad (32)$$

$$\mathbb{B}_2 = (1+Л).\frac{9\bar{\mu}^4}{8(1+\bar{\mu}^2)^2}\lambda_1^2 = (1+Л).\frac{9\bar{\mu}^4 \bar{A}_2 \lambda_1^2}{8} \quad (33)$$

$$\mathbb{B}_3 = \frac{\mathbb{B}_2}{4\lambda_1^2}.\left[(\lambda_1 + 1) + \frac{1}{(1+9\bar{\mu}^2)^2 \bar{A}_2}\right] \quad (34)$$

$$\mathbb{B}_4 = \frac{-3\bar{\mu}^2}{16}\bar{A}_2 \quad (35)$$

$$\overleftrightarrow{\psi}_2 = \sum_{k=1}^{Nk} 3\bar{E}_k \bar{I}_k \bar{L}_k \bar{\mu}^4 S_1^4 + \sum_{j=1}^{N_j} 3\bar{E}_j \bar{I}_j \bar{L}_j \bar{\mu}^4 S_2^4 + \sum_{k=1}^{Nk} 3\bar{G}_k \bar{J}_k \bar{L}_k \bar{\mu}^4 C_1^2 S_1^2 + \sum_{j=1}^{N_j} 3\bar{G}_j J_j \bar{L}_j \bar{\mu}^4 C_2^2 S_2^2 \quad (36)$$

$$\bar{d}_1 = \frac{1}{2(1-\mu^2)} + \overleftrightarrow{\psi}_3 \quad (37)$$

$$\bar{d}_2 = \frac{1}{4} \quad (38)$$

$$\bar{d}_3 = (1+Л)\frac{36\bar{\mu}^4 \bar{A}_2}{32} \quad (39)$$

$$\bar{d}_4 = \frac{9\bar{\mu}^4}{32}(1+Л)\left[\left(\frac{\lambda_1+1}{\lambda_1}\right)\bar{A}_2 + \frac{1}{(9+\bar{\mu}^2)^2}\right] \quad (40)$$

$$\bar{d}_5 = -\frac{3\bar{\mu}^2}{16\lambda_1}(1+Л)\left[\frac{1}{8} - \bar{A}_2\right] \quad (41)$$

$$\overleftrightarrow{\psi}_3 = \sum_{k=1}^{Nk} 3\bar{E}_k \bar{I}_k \bar{L}_k C_1^4 + \sum_{j=1}^{N_j} 3\bar{E}_j \bar{I}_j \bar{L}_j C_2^4 + \sum_{k=1}^{Nk} 3\bar{G}_k \bar{J}_k \bar{L}_k C_1^2 S_1^2 + \sum_{j=1}^{N_j} 3\bar{E}_j \bar{I}_j \bar{L}_j\, C_2^2 S_2^2 \quad (42)$$

Where $\lambda = \frac{b_3^*}{\beta}$ *and* $\lambda_1 = \frac{b_4^*}{b_3^*}$

Eliminating β and b_2^* from Eqns. (3.64), (3.65), (3.66), the following equation was obtained.

$$\mathfrak{M}_1 \Phi^2 + \mathfrak{M}_2 \Phi + \mathfrak{M}_3 = 0 \quad (43)$$

Where

$$\mathfrak{M}_1 = \frac{1}{(1-Л^2)}\left[\frac{\omega_2\omega_3}{\eta_3^2}\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda} - \bar{A}_5\right)^2 + \frac{\eta_2}{\eta_3}\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda}\right)^2 - \left(\frac{\eta_2\omega_3}{\eta_3^2} + \frac{\omega_2}{\eta_3}\right) * \left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda} - \bar{A}_5\right)\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda}\right)\right] \quad (44)$$

$$\mathfrak{M}_2 = -\frac{\bar{\mu}^2 \bar{P}}{(1-Л^2)}\left\{\frac{2\omega_2\omega_3}{\eta_3^2}\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right)\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda} - \bar{A}_5\right) + \frac{2\eta_2}{\eta_3}\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda} - \bar{A}_5\right) - \left(\frac{\eta_2\omega_3}{\eta_3^2} + \frac{\omega_2}{\eta_3}\right)\left[\bar{A}_5^2 + 2\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda}\right)\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right) - \bar{A}_5\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda} + \bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right)\right]\right\} \quad (45)$$

$$\mathfrak{M}_3 = \frac{\bar{\mu}^4 \bar{P}^2}{(1-Л^2)}\left[\frac{\omega_2\omega_3}{\eta_3^2}\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right)^2 + \frac{\eta_2}{\eta_3}\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda} - \bar{A}_5\right)^2 - \left(\frac{\eta_2\omega_3}{\eta_3^2} + \frac{\omega_3}{\eta_3}\right)\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right)\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda} - \bar{A}_5\right)\right] + \frac{{\eta_2}^2\omega_3^2}{\eta_3^2} - \frac{2\omega_2\omega_3\eta_2}{\eta_3} + \omega_2^2 \quad (46)$$

$$\omega_2 = \bar{A}_1\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda}\right) - \mathbb{B}_1 \bar{A}_5 \quad (47)$$

$$\omega_3 = (\bar{A}_2 + \bar{A}_3\lambda + \bar{A}_4\lambda^2)\left(\mathbb{B}_3 + \frac{\mathbb{B}_4}{\lambda}\right) - \mathbb{B}_2 \bar{A}_5 \lambda^2 \quad (48)$$

$$\eta_2 = \bar{A}_1\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right) - \bar{d}_1 \bar{A}_5 \quad (49)$$

$$\eta_3 = (\bar{A}_2 + \bar{A}_3\lambda + \bar{A}_4\lambda^2)\left(\bar{d}_4 + \frac{\bar{d}_5}{\lambda}\right) - (\bar{d}_2 + \bar{d}_3\lambda^2)\bar{A}_5 \quad (50)$$

And

$$\Phi = \bar{\sigma}_b + {}^{3}/_{2}\,\bar{\sigma} \quad (51)$$

Equation (43) is the governing equation for determining the critical buckling stress of an internally pressurized thin cylindrical shell reinforced with angular stiffeners and loaded with eccentric compressive force or bending, where Φ is called minimum stress parameter for stiffened shell under bending and internal pressure.

3.2 Critical Buckling Stress of Internally Pressurized Thin Cylindrical Shell Reinforced with Rings-and-Stringers under uniform Bending

The critical buckling stress of thin cylindrical shell reinforced with stringers and rings that are uniformly spaced, but not far apart was obtained by setting $\gamma_1 = 0^0 (stringers)\ and\ \gamma_2 = 90^0\ (rings)$ respectively.

Hence,$C_1 = \cos\gamma_1 = \cos 0^0 = 1;\ S_1 = \sin\gamma_1 = \sin 0^0 = 0;$ C2=cosγ2=cos*900*=0 and S2=sinγ2=sin*900=1*

The solution of Eqn. (18b) after minimization for the condition under consideration yielded

$$\frac{\bar{\emptyset}_1\beta}{1-Л} = \overleftrightarrow{A_1} + \beta^2\left(\overleftrightarrow{A_2} + \overleftrightarrow{A_3}\lambda + \overleftrightarrow{A_4}\lambda^2\right) + ƀ_2^{*2}\overleftrightarrow{A_5} \qquad (52)$$

$$\frac{\bar{\emptyset}_2\beta}{1-Л} = \overleftrightarrow{\mathbb{B}_1} + \beta^2\overleftrightarrow{\mathbb{B}_2}\lambda^2 + ƀ_2^{*2}\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda}\right) \qquad (53)$$

$$\bar{\emptyset}_3\frac{\beta}{1-Л} = \bar{\bar{d}}_1 + \left(\bar{\bar{d}}_2 + \bar{\bar{d}}_3\lambda^2\right)\beta^2 + ƀ_2^{*2}\left(\bar{\bar{d}}_4 + \frac{\bar{\bar{d}}_5}{\lambda}\right) \qquad (54)$$

The notations used for the above equation as derived were defined as follows; in Eqn. (52)

$$\bar{\emptyset}_1 = \frac{3}{2}\bar{\sigma} + \bar{\sigma}_b - \frac{\bar{\mu}^2 3\bar{P}}{2} = \frac{3}{2}\bar{\sigma} + \bar{\sigma}_b + \bar{\emptyset}_2 \qquad (55)$$

$$\overleftrightarrow{A_1} = \frac{(1+\bar{\mu}^2)^2}{8(1-\mu^2)} + \overleftrightarrow{\bar{\psi}_1} \qquad (56)$$

$$\overleftrightarrow{A_2} = \bar{A}_2 = \frac{1}{(1+\bar{\mu}^2)^2} \qquad (57)$$

$$\overleftrightarrow{A_3} = \bar{A}_3 = -\left[\frac{3(2+Л)(1+\lambda_1)\bar{\mu}^2}{2(1+\bar{\mu}^2)^2} + \frac{3}{8}\bar{\mu}^2\lambda_1\right] = -\frac{3}{2}\left[(2+Л)(1+\lambda_1)\bar{\mu}^2\bar{A}_2 + \frac{1}{4}\bar{\mu}^2\lambda_1\right] \qquad (58)$$

$$\overleftrightarrow{A_4} = \bar{A}_4 = (1+Л)\left\{\frac{9(1+\lambda_1)^2\bar{\mu}^4}{4(1+\bar{\mu}^2)^2} + \frac{9\bar{\mu}^4}{4}\left[\frac{\lambda_1^2}{(9+\bar{\mu}^2)^2} + \frac{1}{(1+9\bar{\mu}^2)^2}\right]\right\} = (1+Л)\left\{\frac{9(1+\lambda_1)^2\bar{\mu}^4\bar{A}_2}{4} + \frac{9\bar{\mu}^4}{4}\left[\frac{\lambda_1^2}{(9+\bar{\mu}^2)^2} + \frac{1}{(1+9\bar{\mu}^2)^2}\right]\right\} \qquad (59)$$

$$\overleftrightarrow{A_5} = \bar{A}_5 = (1+Л).\frac{9(1+\bar{\mu}^4)}{256} \qquad (60)$$

$$\overleftrightarrow{\bar{\psi}_1} = \sum_{k=1}^{Nk}\frac{3\bar{E}_k\bar{I}_k\bar{L}_k}{4} + \sum_{j=1}^{N_j}\frac{3\bar{E}_j\bar{I}_j\bar{L}_j\bar{\mu}^4}{4} + \sum_{k=1}^{Nk}\frac{3\bar{G}_k\bar{J}_k\bar{L}_k\bar{\mu}^2}{4} + \sum_{j=1}^{N_j}\frac{3\bar{G}_j\bar{J}_j\bar{L}_j\bar{\mu}^4}{4} \qquad (61)$$

The notations used in Eqn. (53) as derived were defined as follows;

$$\bar{\emptyset}_2 = -\frac{3\bar{\mu}^2\bar{P}}{2} \qquad (62)$$

$$\overleftrightarrow{\mathbb{B}_1} = \frac{\bar{\mu}^4}{2(1-\mu^2)} + \overleftrightarrow{\bar{\psi}_2} \qquad (63)$$

$$\overleftrightarrow{\mathbb{B}_2} = (1+Л).\frac{9\bar{\mu}^4}{8(1+\bar{\mu}^2)^2}\lambda_1^2 = (1+Л).\frac{9\bar{\mu}^4\bar{A}_2\lambda_1^2}{8} \qquad (64)$$

$$\overleftrightarrow{\mathbb{B}_3} = \mathbb{B}_3 = (1+Л)\left[\frac{9\bar{\mu}^4}{32(1+\bar{\mu}^2)^2}(1+\lambda_1) + \frac{9\bar{\mu}^4}{32(1+9\bar{\mu}^2)^2}\right] = \frac{\mathbb{B}_2}{4\lambda_1^2}.\left[(\lambda_1+1) + \frac{1}{(1+9\bar{\mu}^2)^2\bar{A}_2}\right] \qquad (65)$$

$$\overleftrightarrow{\mathbb{B}_4} = \mathbb{B}_4 = \frac{-3\bar{\mu}^2}{16(1+\bar{\mu}^2)^2} = \frac{-3\bar{\mu}^2}{16}\bar{A}_2 \qquad (66)$$

$$\overleftrightarrow{\bar{\psi}_2} = \sum_{j=1}^{N_j}3\bar{E}_j\bar{I}_j\bar{L}_j\bar{\mu}^4 \qquad (67)$$

The notations used in Eqn. (54) as derived were defined as follows;

$$\bar{\emptyset}_3 = \sigma_b + \frac{3}{2}\bar{\sigma} \qquad (68)$$

$$\bar{\bar{d}}_1 = \frac{1}{2(1-\mu^2)} + \overleftrightarrow{\bar{\psi}_3} \qquad (69)$$

$$\bar{\bar{d}}_2 = \bar{d}_2 = \frac{1}{4} \qquad (70)$$

$$\bar{\bar{d}}_3 = \bar{d}_3 = (1+Л)\frac{36\bar{\mu}^4}{32(1+\bar{\mu}^2)^2} = (1+Л)\frac{36\bar{\mu}^4\bar{A}_2}{32} \qquad (71)$$

$$\bar{\bar{d}}_4 = \bar{d}_4 = \frac{9\bar{\mu}^4}{32}(1+Л)\left[\left(1+\frac{1}{\lambda_1}\right)\frac{1}{(1+\bar{\mu}^2)^2} + \frac{1}{(9+\bar{\mu}^2)^2}\right] = \frac{9\bar{\mu}^4}{32}(1+Л)\left[\left(\frac{\lambda_1+1}{\lambda_1}\right)\bar{A}_2 + \frac{1}{(9+\bar{\mu}^2)^2}\right] \qquad (72)$$

$$\bar{\bar{d}}_5 = \bar{d}_5 = -(1+Л)\frac{3\bar{\mu}^2}{128\lambda_1} - \frac{3\bar{\mu}^2}{16(1+\bar{\mu}^2)^2\lambda_1} = -(1+Л)\frac{3\bar{\mu}^2}{16\lambda_1}\left[\frac{1}{8} - \frac{1}{(1+\bar{\mu}^2)^2}\right] = -\frac{3\bar{\mu}^2}{16\lambda_1}(1+Л)\left[\frac{1}{8} - \bar{A}_2\right] \qquad (73)$$

$$\overleftrightarrow{\bar{\psi}_3} = \sum_{k=1}^{Nk}3\bar{E}_k\bar{I}_k\bar{L}_k \qquad (74)$$

Recall that $\lambda = \frac{ƀ_3^*}{\beta}$ *and* $\lambda_1 = \frac{ƀ_4^*}{ƀ_3^*}$ (75)

Eliminating β and $ƀ_2^*$ from Eqns. (52), (53),and (54), the following equation was obtained.

$$\overleftrightarrow{\mathfrak{M}}_1\Phi^2 + \overleftrightarrow{\mathfrak{M}}_2\Phi + \overleftrightarrow{\mathfrak{M}}_3 = 0 \qquad (76)$$

where

$$\overleftrightarrow{\mathfrak{M}}_1 = \frac{1}{(1-Л^2)}\left[\frac{\bar{\varpi}_2\bar{\varpi}_3}{\bar{\mathfrak{y}}_3^2}\left(\bar{\bar{d}}_4 + \frac{\bar{\bar{d}}_5}{\lambda} - \overleftrightarrow{A_5}\right)^2 + \frac{\bar{\mathfrak{y}}_2}{\bar{\mathfrak{y}}_3}\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda}\right)^2 - \left(\frac{\bar{\mathfrak{y}}_2\bar{\varpi}_3}{\bar{\mathfrak{y}}_3^2} + \frac{\bar{\varpi}_2}{\bar{\mathfrak{y}}_3}\right) * \left(\bar{\bar{d}}_4 + \frac{\bar{\bar{d}}_5}{\lambda} - \overleftrightarrow{A_5}\right)\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda}\right)\right] \qquad (77)$$

$$\overleftrightarrow{\mathfrak{M}}_2 = -\frac{\bar{\mu}^2\bar{P}}{(1-Л^2)}\left\{\frac{2\bar{\varpi}_2\bar{\varpi}_3}{\bar{\mathfrak{y}}_3^2}(\bar{\bar{d}}_4 + \frac{\bar{\bar{d}}_5}{\lambda})\left(\bar{\bar{d}}_4 + \frac{\bar{\bar{d}}_5}{\lambda} - \overleftrightarrow{A_5}\right) + \frac{2\bar{\mathfrak{y}}_2}{\bar{\mathfrak{y}}_3}\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda} - \overleftrightarrow{A_5}\right) - \left(\frac{\bar{\mathfrak{y}}_2\bar{\varpi}_3}{\bar{\mathfrak{y}}_3^2} + \frac{\bar{\varpi}_2}{\bar{\mathfrak{y}}_3}\right)\left[\overleftrightarrow{A_5}^2 + 2\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda}\right)\left(\bar{\bar{d}}_4 + \frac{\bar{\bar{d}}_5}{\lambda}\right) - \overleftrightarrow{A_5}\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda} + \bar{\bar{d}}_4 + \frac{\bar{\bar{d}}_5}{\lambda}\right)\right]\right\} \qquad (78)$$

$$\overleftrightarrow{\mathfrak{M}}_3 = \frac{\bar{\mu}^4 \bar{P}^2}{(1-Л^2)}\left[\frac{\bar{\varpi}_2\bar{\varpi}_3}{\bar{\mathfrak{y}}_3^2}\left(\bar{\bar{\mathrm{d}}}_4 + \frac{\bar{\bar{\mathrm{d}}}_5}{\lambda}\right)^2 + \frac{\bar{\mathfrak{y}}_2}{\bar{\mathfrak{y}}_3}\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda} - \overleftrightarrow{A_5}\right)^2 - \left(\frac{\bar{\mathfrak{y}}_2\bar{\varpi}_3}{\bar{\mathfrak{y}}_3^2} + \frac{\bar{\varpi}_3}{\bar{\mathfrak{y}}_3}\right)\left(\bar{\bar{\mathrm{d}}}_4 + \frac{\bar{\bar{\mathrm{d}}}_5}{\lambda}\right)\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda} - \overleftrightarrow{A_5}\right)\right] + \frac{\bar{\mathfrak{y}}_2{}^2\bar{\varpi}_3^2}{\bar{\mathfrak{y}}_3^2} - \frac{2\bar{\varpi}_2\bar{\varpi}_3\bar{\mathfrak{y}}_2}{\bar{\mathfrak{y}}_3} + \bar{\varpi}_2^2 \quad (79)$$

$$\bar{\varpi}_2 = \overleftrightarrow{A_1}\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda}\right) - \overleftrightarrow{\mathbb{B}_1}\overleftrightarrow{A_5} \quad (80)$$

$$\bar{\varpi}_3 = (\overleftrightarrow{A_2} + \overleftrightarrow{A_3}\lambda + \overleftrightarrow{A_4}\lambda^2)\left(\overleftrightarrow{\mathbb{B}_3} + \frac{\overleftrightarrow{\mathbb{B}_4}}{\lambda}\right) - \overleftrightarrow{\mathbb{B}_2}\overleftrightarrow{A_5}\lambda^2 \quad (81)$$

$$\bar{\mathfrak{y}}_2 = \overleftrightarrow{A_1}\left(\bar{\bar{\mathrm{d}}}_4 + \frac{\bar{\bar{\mathrm{d}}}_5}{\lambda}\right) - \bar{\bar{\mathrm{d}}}_1\overleftrightarrow{A_5} \quad (82)$$

$$\bar{\mathfrak{y}}_3 = (\overleftrightarrow{A_2} + \overleftrightarrow{A_3}\lambda + \overleftrightarrow{A_4}\lambda^2)\left(\bar{\bar{\mathrm{d}}}_4 + \frac{\bar{\bar{\mathrm{d}}}_5}{\lambda}\right) - (\bar{\bar{\mathrm{d}}}_2 + \bar{\bar{\mathrm{d}}}_3\lambda^2)\overleftrightarrow{A_5} \quad (83)$$

And

where $\Phi = \bar{\sigma}_b + {}^3/_2\,\bar{\sigma}$ (84)

Equation (76) is the governing equation for determining the critical buckling stress of an internally pressurized thin cylindrical shell reinforced with rings-and-stringers due to bending

4.0 RESULTS AND DISCUSSIONS

4.1 Results

NUMERICAL EXAMPLES

The numerical analysis of this type of cylindrical shell was done by taking the following assumptions: $\bar{E}_k\bar{I}_k\bar{L}_k = \bar{E}_j\bar{I}_j\bar{L}_j, \bar{G}_k\bar{J}_k\bar{L}_k = \bar{G}_j\bar{J}_j\bar{L}_j$, $\curlyvee_1 = \curlyvee_2 = \curlyvee\,(for\,\curlyvee = 10^0, 20^0, 30^0, 40^0, 45^0, 50^0, 60^0)$, $\lambda = \lambda_1$, $m = 5$, $\bar{\mu} = 1$, $h = 0.05\,metre$, $\bar{P} = 2\,and\,R = 2\,metres$. Using the governing equation in Eqns (43) and (76) the notations described from Eqn(44) to Eqn (50) and that described from Eqn(77) to Eqn (83), the following data shown in Table 1 obtained for different imperfect ratio, Л

IMPERFECT RATIO, Л	BUCKLING STRESS PARAMETER, Φ OF STIFFENERS AT DIFFERENT ANGLES DUE TO UNIFORM BENDING							Φ FOR RINGS-AND-STRINGERS
	10^0	20^0	30^0	40^0	45^0	50^0	60^0	
0.1	7.6307	4.6271	1.4266	0.4038	0.3430	0.4785	1.2589	0.3929
0.2	8.1084	5.3111	1.6910	0.4387	0.3665	0.5358	1.5150	0.4474
0.3	8.4625	5.9047	1.9723	0.4819	0.3913	0.5865	1.7508	0.4879
0.4	8.6674	6.3654	2.2466	0.5316	0.4167	0.6268	1.9430	0.5131
0.5	8.6951	6.6554	2.4860	0.5863	0.4425	0.6552	2.0576	0.5207
0.6	8.5097	6.7345	2.6572	0.6422	0.4684	0.6573	2.0488	0.5073
0.7	8.0586	6.5516	2.7201	0.7031	0.4943	0.6360	1.8640	0.4690
0.8	7.2535	6.0284	2.6262	0.7602	0.5202	0.5811	1.4580	0.4014
0.9	5.9127	5.0181	2.3144	0.8118	0.5461	0.4846	0.8131	0.2997

4.2 Discussion of Results

The data in Table 4.1 shows that the worst critical buckling stress of internally pressurized thin cylindrical shell considered in this research occurs when the stiffeners are inclined at 45^0 at imperfect ratio of 0.1 While, the maximum buckling stress occurs when the stiffeners are inclined at 10^0 at imperfect ratio of 0.5 .Also, the buckling stress for the cylinders reinforced with rings-and-stringers considered in this work are averagely greater than those reinforced with stiffeners inclined at 45^0, but less than those reinforced with stiffeners inclined at 10^0, 20^0, 30^0, 50^0 and 60^0 respectively. Thus, rings-and-stringers are more effective than inclined stiffeners at 45^0, but less effective than stiffeners inclined at 10^0, 20^0, 30^0, 50^0 and 60^0 respectively.

5.0 CONCLUSION

With reference to the results obtained in this research, engineers designing cylindrical shell structures with the aim of providing resistance to buckling would be able to select suitable stiffeners for the structure under uniform bending.

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